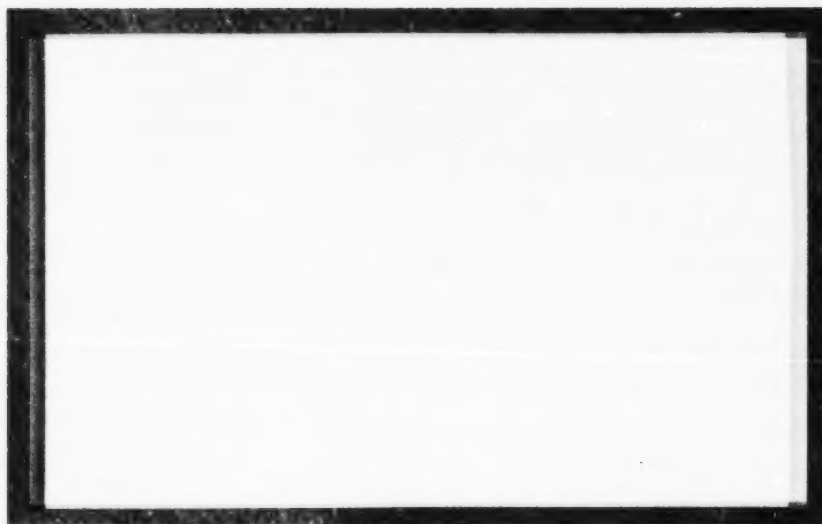


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ANALYSIS OF PLATOON DISPERSION
WITH RESPECT TO TRAFFIC VOLUME

by

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ABSTRACT

Platoon dispersion depends on friction. The authors of the present article hypothesize that this friction can be external and internal and that the internal friction depends on traffic density and volume. Platoon dispersion should consequently be influenced by traffic density and volume. This hypothetical relationship was confirmed by limited field observations and simulations. Platoon dispersion increases with increasing volume up to a maximum value, which is observed at lower volumes than capacity. Dispersion decreases with further increasing volume and becomes nearly zero at volumes near capacity. If further studies confirm the observed relationship, platoon dispersion could be calculated directly from densities or volumes and travel times, applied to Robertson's model on a link by link basis, thus improving the results of traffic signal coordination models.

Key words: Traffic, platoon dispersion, signal systems, Robertson's model.

RÉSUMÉ

La dispersion des pelotons dépend du "frottement". Les auteurs du présent article font l'hypothèse que ce frottement a une composante interne et une composante externe. Le frottement interne dépend de la densité et du débit de circulation. La dispersion des pelotons devrait donc, par conséquent, être influencée également par ces variables. Cette relation hypothétique a été confirmée par des observations et des simulations sur l'ordinateur. La dispersion des pelotons augmente avec le débit jusqu'à une valeur maximale; celle-ci est atteinte à un débit plus faible que la capacité de la route. Elle décroît ensuite jusqu'à zéro lorsque le débit atteint la capacité. On pourrait directement calculer, à l'aide du modèle de Robertson, la dispersion des pelotons à partir des densités et des débits, si cette relation est confirmée par des études subséquentes. Une telle relation pourrait être appliquée individuellement pour chaque lien du réseau, ce qui améliorerait les résultats des modèles de coordination des feux.

Mots clés: Circulation, dispersion des pelotons, coordination des feux, modèle de Robertson.

ANALYSIS OF PLATOON DISPERSION WITH RESPECT TO TRAFFIC VOLUME

INTRODUCTION

Traffic signals have the effect of grouping vehicles into platoons which leave the intersection under saturated flow conditions. As the platoon moves down the arterial it tends to spread out over time and space. This phenomenon of platoon dispersion is an important factor in traffic signal coordination. Robertson's model allows one to calculate the histogram of a platoon at different points downstream of an intersection. The basic equation which determines the degree of dispersion is:

$$F = \frac{1}{1 + \alpha \beta t} = \frac{1}{1 + K t}$$

The value of K (a capital letter is used here, to avoid confusion with the density k) in this equation depends on the type of road under study and on the degree of friction. McCoy et al. (1983) (1) describe platoon dispersion in lower friction environments and the TRANSYT 7F manual allows one to choose a K value depending on several degrees of friction ranging from low to high as in figure 1.

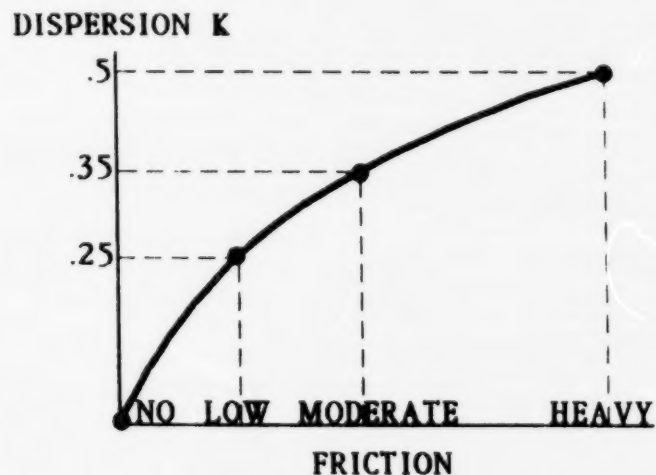


Figure 1. Platoon dispersion as a function of friction (TRANSYT 7F)

This figure shows that platoon dispersion increases with friction. It would be useful to have a more quantitative measure of friction that could take explicitly into account an independent variable like service volume or volume divided by saturation flow. Service volumes or saturation flows would be established considering the many independent variables such as corridor width, percentage of heavy vehicles and so on, and would be particular to a given arterial type. If friction could then be explained by service volumes, the basic dispersion equation would become a function of service volumes and travel times. Platoon dispersion could then be integrated on a link basis into the traffic signal coordination model.

The aim of the article is to describe the concepts of such a relationship based on field observations, on microscopic simulations and on theoretical considerations.

The concept and some theoretical considerations

One can easily observe that platoon dispersion is a function of the physical features of the highway facility and of the interaction between vehicles in the traffic stream. The friction concept introduced by May (2) in 1959 is very useful in the analysis and explanation of platoon dispersion. Friction results from two phenomena: internal friction between moving vehicles and external friction related to roadway features. External and internal friction are implicitly considered in the concept of level of service, since it takes into account such variables as number of lanes and their widths, vertical and horizontal alignment and the location of the arterial within the city.

The internal friction reflects the individual driver's freedom to maneuver, his comfort, his safety and the greater the traffic volumes the more adverse the effects of friction. Conceptually, when volume and density are zero, friction is zero too and increases with increasing volume and density up to a certain maximal value, which occurs, at a lower volume than the maximal volume (level of

service "E"). At further volume increases, friction diminishes and approaches zero, which happens at capacity when every vehicle is moving at nearly the same speed. Thereafter, friction again increases with increasing density and should be maximal (at least theoretically) at jam density. Figure 2 illustrates this conceptual relationship.

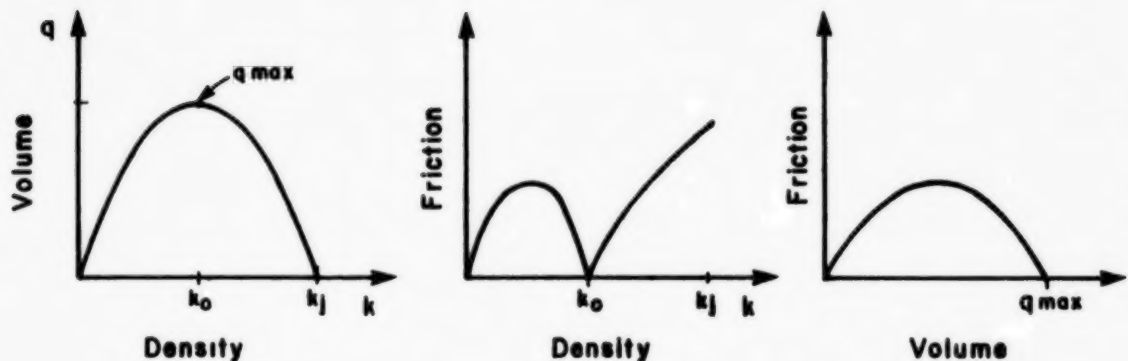


Figure 2. Conceptual relationship between friction, density and volume.

The purpose of the following paragraphs is to see if there is a rational explanation of this interrelationship by relating it to the different phases of driver behavior as volume and density increase.

May (1963) (3) and Leutzbach (1972) (4), for example, distinguish three zones of different driver behavior. May identifies the three zones on the volume-density curve, they are: a zone of constant speed, a second of constant volumes and a third of constant rate of change of volumes with density.

In the following an explanation for this division into three zones is attempted and at the same time a rational explanation for friction and corresponding platoon dispersion is given.

In order to analyze driver behavior with respect to volume changes, let us study the volume-spacing relationship which is illustrated in figure 3.

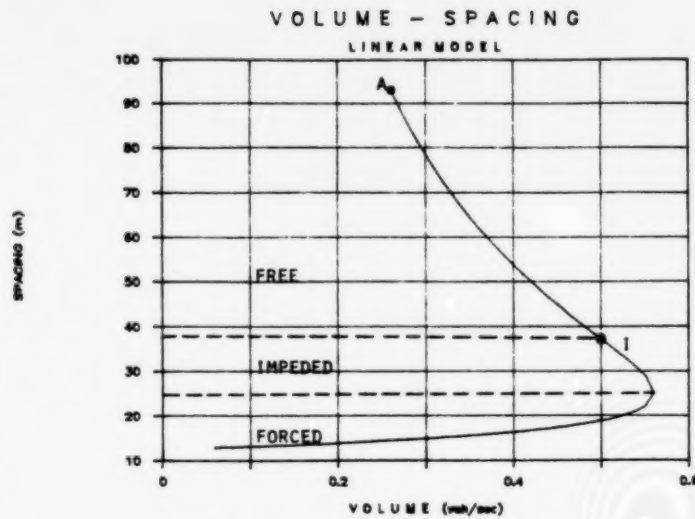


Figure 3. Volume spacing relationship (linear model).

For the purpose of example, this figure represents the linear macroscopic traffic model with a free speed of $v_f = 28$ (m/s) and a jam density of $k_j = 0.08$ (veh/m). Attentive analysis of this figure reveals that there is a point of inflection in the curve at a certain spacing. This means that the driver changes behavior at this point. The situation becomes much clearer when one analyzes the rate of change of spacing with respect to a uniform increase of volume. This relation is shown in figure 4.

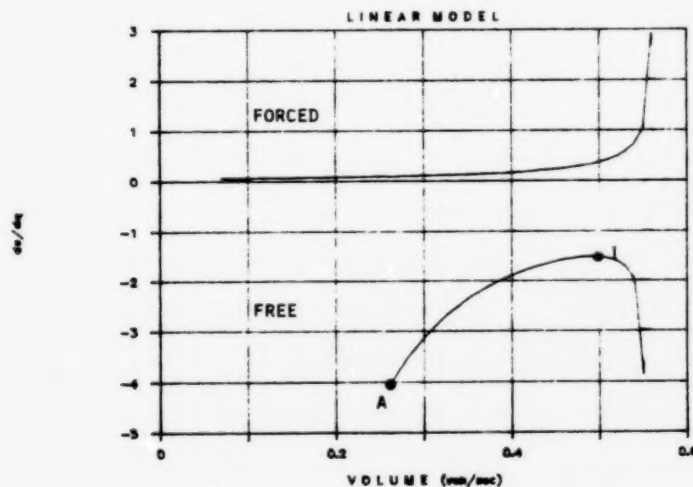


Figure 4. Rate of change of spacing vs. volume.

Starting from a point A on the free flow side of the curve the rate of change of spacing decreases faster and faster with increasing volume up to a point, where the rate of change remains nearly constant (at the point of inflection I). For more clarity and in order to better understand the changing behavior of the driver let us consider figures 5 and 6 which represent the rate of change of speed and headway with respect to volume.

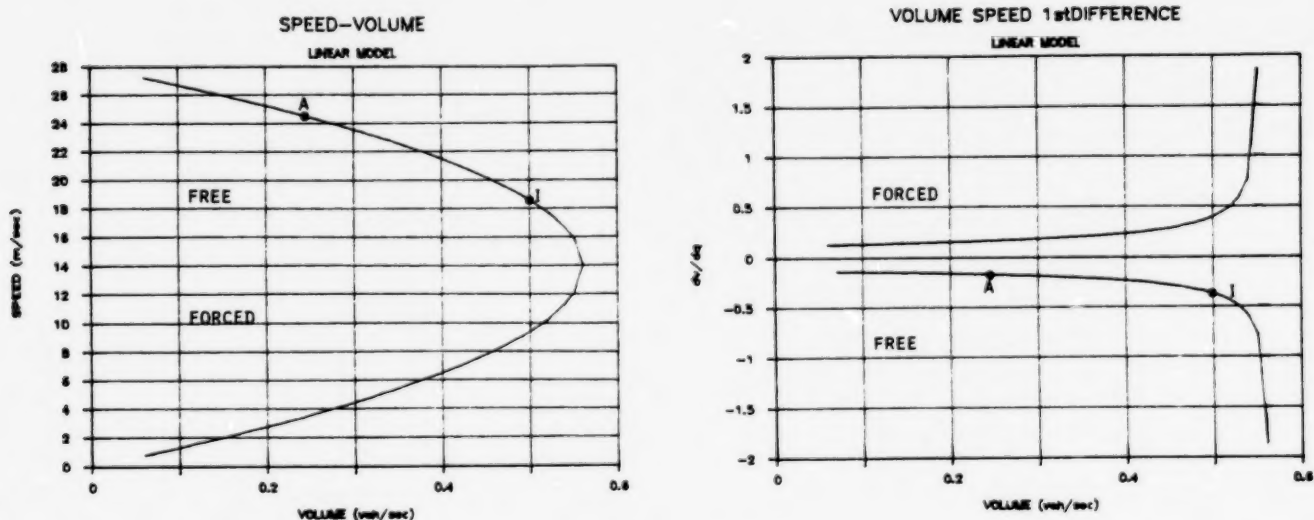


Figure 5. Speed- volume relationship and rate of change of speed to volume.

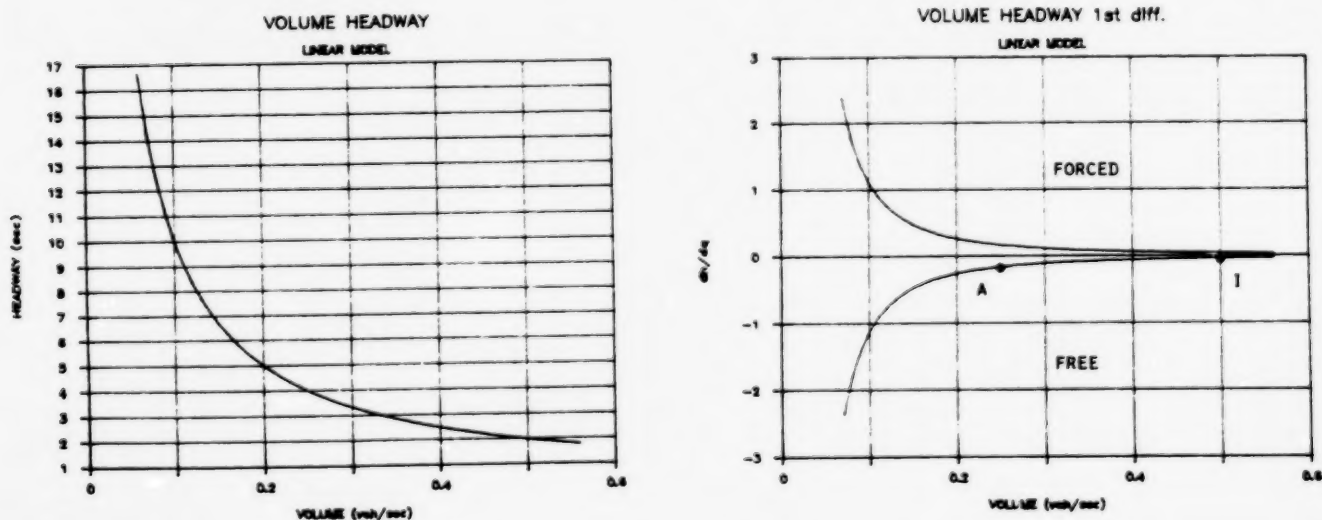


Figure 6. Headway- volume relationship and rate of change of headway to volume.

Near the point I, the rate of change of spacings is constant while the rate of change of speeds is increasing and the rate of change of headways is nearly constant. Since headways remain nearly constant at volumes near the capacity, spacings and speeds will change rapidly when volumes increase beyond the point of inflection.

At volumes higher than those observed at the point of inflection (q_i) drivers have a different behavior with respect to the car they follow than at lower volumes. On figure 4 one can distinguish three zones, the zone of free flow, of impeded flow and of forced flow.

This change in driver behavior happens to influence platoon dispersion. Dispersion increases with increasing volumes up to or near the point of inflection and then decreases to a low, near zero value, at capacity. The notion of platoon dispersion at densities higher than those observed at capacity seem to be meaningless.

Mathematical investigation into the point of changing behavior

The location of the point of inflection on the volume density curve was studied in more detail for the macroscopic traffic stream models defined by Drew (1968) (5). In his notation $n=-1$ represents the exponential model, $n=0$ the parabolic model and $n=1$ the linear model.

The formulae relating spacing s to volume q for these models are:

$$n=-1 \quad q = \frac{1}{s} u_m [\ln(sk_p)]$$

$$\frac{dq}{ds} = -\frac{1}{s^2} u_m [1 - \ln(sk_p)]$$

$$\frac{dq}{ds^2} = \frac{1}{s^3} u_m [2 \ln(sk_j) - 3]$$

and for $n > -1$:

$$q = \frac{u_m}{s} [1 - (1/sk_j)^{(n+1)/2}]$$

$$\frac{dq}{ds} = u_m \left[\frac{(n+3)}{2} s^{-(n+5)/2} (k_j)^{-(n+1)/2} - s^{-2} \right]$$

$$\frac{dq}{ds} = u_m \left[2 s^{-3} - \frac{(n^2 + 8n + 15)}{4} s^{-(n+5)/2} (k_j)^{-(n+1)/2} \right]$$

Table 1 gives a comparison between the spacings s_i at the point of inflection and the spacing s_m at maximal volume as a function of the jam spacing s_j . Spacings at the point of inflection or of changing driver behavior are 1.5 times longer than the spacings at capacity and the volume is 10% less than the maximum volume.

Interestingly, the point of inflection is very near or at the same point as the maximum kinetic energy described by Drew (1965) (6). Rewriting the energy equation as a function of spacing we have:

$n = -1$

$$E = \frac{(u_m)^2}{s} [\ln(sk_j)]^2$$

$n > -1$

$$E = \frac{(u_m)^2}{s} [1 - 2 (sk_j)^{-(n+1)/2} + (sk_j)^{-(n+1)}]$$

Model	n	Spacing at point of inflection	max. q	s_i/s_m	Volume at point of inflection	max. q	q_i/q_m
exponential	$n=-1$	$4.48s_j$	$2.72s_j$	1.65	$0.33u_mk_j$	$0.37u_mk_j$	0.91
parabolic	$n=0$	$3.52s_j$	$2.25s_j$	1.56	$0.13u_mk_j$	$0.15u_mk_j$	0.9
linear	$n=1$	$3s_j$	$2s_j$	1.5	$0.22u_mk_j$	$0.25u_mk_j$	0.9

Table 1. Flow and spacing at point of inflection and at capacity.

The curve relating spacing to kinetic energy is given, as an example, for the linear model in figure 7.

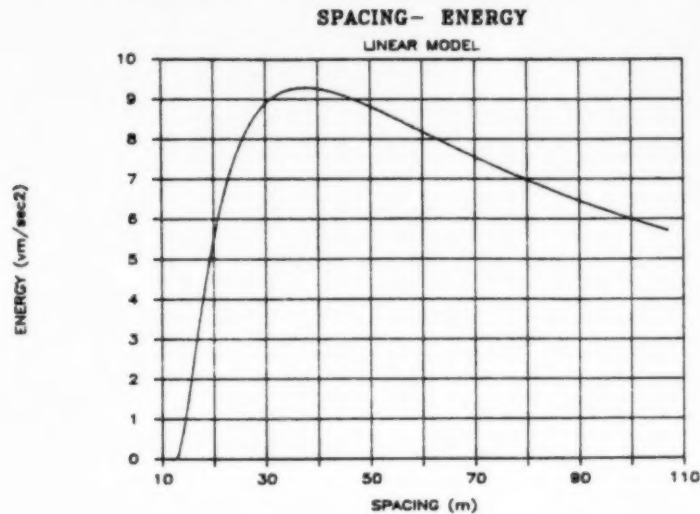


Figure 7. Spacing vs. kinetic energy for the linear model.

A comparison of spacings at the point of maximum kinetic energy and the at the point of inflection is given in table 2.

Model	n	Spacing at point of inflection	max. E	s_i/s_E
exponential	$n=-1$	$4.48s_j$	$7.39s_j$	0.61
parabolic	$n=0$	$3.52s_j$	$4s_j$	0.88
linear	$n=1$	$3s_j$	$3s_j$	1

Table 2. Comparison of spacings at the point of inflection and at the point of maximum kinetic energy.

Drew et al. (1965) (6) state that the kinetic energy of a traffic stream is equal to the sum of energies of its constituent particles and will be a maximum when internal friction caused by vehicular interaction is a minimum. In the case of the linear model the point of inflection and maximum energy or theoretical minimum of internal friction fall together. One can conclude that there is a point where driver behavior changes with respect to spacing and this point

occurs at lower densities and at lower volumes than the capacity. When the volume approaches a certain value, drivers reduce less and less their spacings as a reaction to a unit increase of volume. At the point of inflection this tendency is sharply reversed. The actual location of this point of change of driver behavior depends on the type of facility under study, and on the model chosen. One might compare this point of inflection to the transition between laminar and turbulent flow in hydraulics.

Other criteria for representing internal friction

Lee et al. (1973) (7) developed a criterion for internal energy or friction based on observations. They found that acceleration noise, as proposed by Drew, does not accurately represent internal friction or the internal energy of the traffic stream. Acceleration noise, at least in their observations, did not show a clear relationship to increasing density. They propose, as a good measure for internal friction, the coefficient of variation of speed σ/v , also called the constant of diffusion. As spacings decrease, this measure increases, up to a point (the point of maximum energy or the point of inflection) and decreases then to nearly zero at capacity. Figure 8 shows this indicator for the values found by Lee et al. as a function of density.

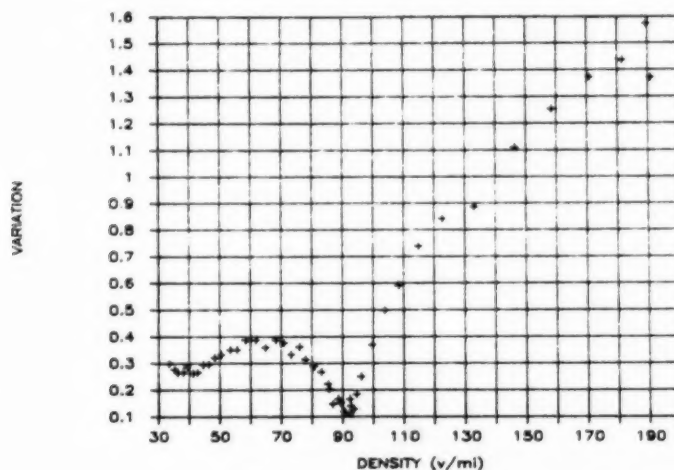


Figure 8. Diffusion constant vs. density. (Lee et al. (1973)).

Even if in their paper there was no indication about the volume density relationship, one can see that the relationship follows the one postulated in the preceding paragraphs for the relation between dispersion and volume.

The measure of friction developed by Helly et al. (1967) (8) is the mean velocity gradient, which represents the acceleration noise divided by the average velocity. The measure seems to have some merits but there are not enough observations available to confirm this.

In the light of what was said, one can hypothesize that dispersion depends on service volumes, or in a simpler way, on volumes divided by the saturation flow. In order to confirm the hypothesis stated in the preceding paragraphs a field study was carried out on two straight and level 3-lane urban arterials, located in an intermediate type area within the city of Montreal.

The field study

The arterials were chosen so as to minimize interferences and marginal and intersectional friction. The distance to the downstream intersection was around 800 meters so that the traffic flow be unaffected by the downstream traffic control devices. The observations were made at the intersection (stop-line plus the width of the cross-street) at 100 meters and at 200 meters from that point. Both arterials had high volumes during peak-hours with platoons ranging from 16 to 90 vehicles, with the small platoons observed at the beginning and at the end of the peak-hour. Approximately 150 platoons were observed at each arterial. These observations were carried out with three hand-held micro-computers (Tandy 200) with external disk drive, whose internal clocks were synchronized. A program written in BASIC allowed the identification of buses, trucks and passenger cars. Only five of the keys on the keyboard were activated and were identified by stickers as B, T, P and CB and CE, which means bus, truck, passenger car and

beginning and ending of the cycle. The data on the type of vehicle, its passage time and the beginning and the end of the cycle were stored on the portable disk drive. This procedure was extremely efficient. Only 2% of all platoons had to be eliminated in the validation phase and this was mostly due to data saving problems. The commercially available hand-held computers with standard keyboard can be programmed for any data-acquisition problem by means of simple BASIC programs (see Baass (1987) (9) and Bonsall (1987) (10)). Even with a small memory of 32K, as was the case with the Tandy 200, this new technology contributes a completely new and efficient way of traffic data acquisition. The data were then transferred easily into an IBM AT where they were treated directly with help of a spread-sheet (SYMPHONY). Table 3 shows how the validated platoons were classified into groups of similar sizes and average platoons were derived for each platoon size. Figure 9 shows such a platoon at 100 and at 200 meters from the stop-line.

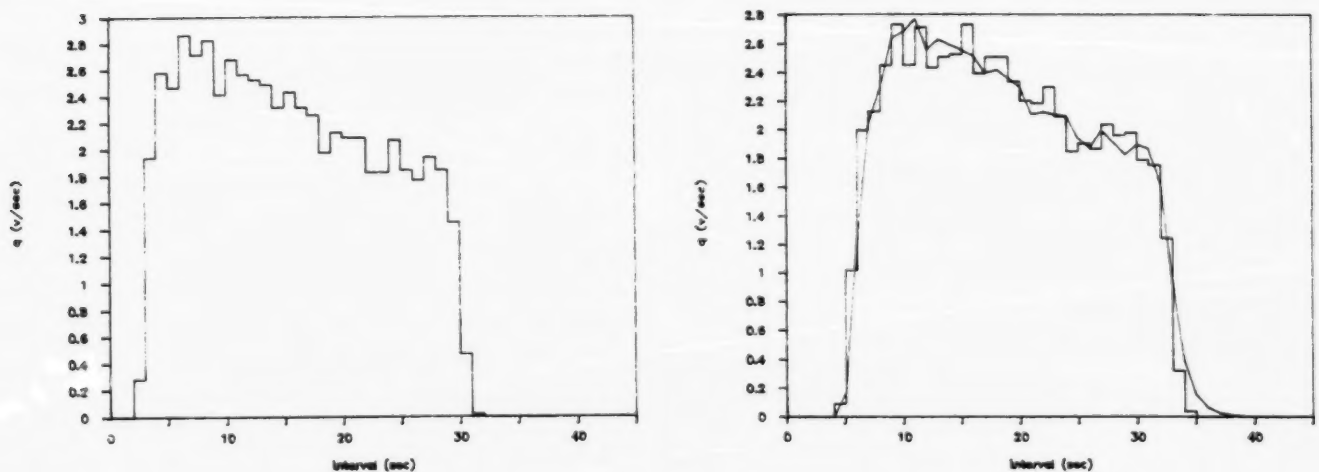


Figure 9. Histogram of a large platoon (68) at station 100 and 200 m.

no of platoons	Papineau Ave		Laurentian Bld	
observed	338		130	
eliminated	7		4	
	platoon size	no of platoons	platoon size	no of platoons
	16	22		
	19	54		
	27	33		
	36	60		
	43	13	46	7
	50	13	55	12
	54	26	60	17
	61	53	67	19
	65	20	73	27
	68	27	75	12
	74	10	78	18
	76	7	81	18

Table 3. Number of observations at the two study locations.

A special procedure designed on the spread- sheet was then used to simulate the platoon dispersion based on the Robertson model for different trial values of K. The parameter β was considered as a constant value of 0.8 in all calibrations due to the complexity of a dual calibration of α and β at the same time (see Axhausen et al. (1987) (11)). The best fitting K was obtained for each average platoon using an indicator similar to the χ^2 by comparing observed and simulated platoon histograms. Thereby the best K values remained nearly constant for all stations.

Papineau Ave		Laurentian Bld	
platoon size	best k	platoon size	best K
16	0.17		
19	0.325		
27	0.35		
36	0.405		
43	0.39	46	0.13
50	0.33	55	0.18
54	0.325	60	0.16
61	0.315	67	0.17
65	0.31	73	0.13
68	0.32	75	0.13
74	0.26	78	0.1
76	0.255	81	0.13

Table 4. Results of the calibration of k in the model of Robertson.

The best K values which were obtained in this way are given in table 4 and the relation between K and the platoon size are given in figure 10. Lefebvre (1987) (12) describes the experiment and the calibration procedure.

Calculating the saturation flow or level of service E at the study sites and using the cycle lengths observed permitted the drawing of figure 11, which gives the best K with relation to the volume/capacity ratio.

In both the observed cases, the observations and calibrated K values came close to the hypothesized relationship. Further analysis of the data is necessary to explain the relation between cycle duration, platoon size and platoon dispersion K .

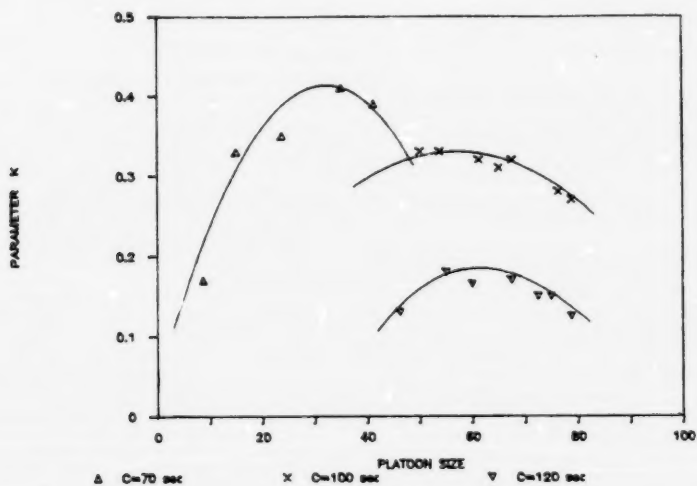


Figure 10. Parameter K for the two study sites in relation to platoon size.

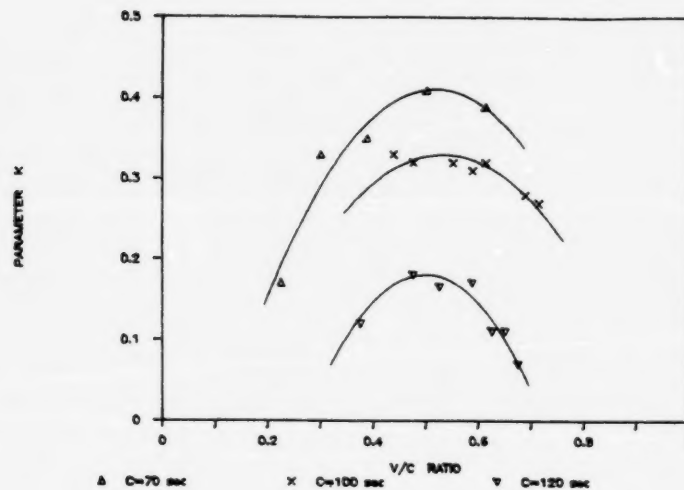


Figure 11. Parameter k for the two study sites in relation to v/c ratio.

The microscopic simulation

Microscopic simulations were carried out on a hypothetical 3-lane arterial in order to further verify the postulated relationship. This arterial was chosen to be different from the arterial streets on which the field study was conducted. Manar (1987) (13) developed a microscopic traffic simulation model based on the car following model described by Fox and Lehman (1967) (14). He introduced into the existing program multi-lane simulation with lane changing behavior (see Seddon (1971) (15)) and increased the platoon sizes to up to 110 vehicles on three lanes.

Different sizes of average platoons were obtained with the simulation model. The histograms describing the platoons were calculated for the average platoons at 0, 100 and 200 meters from the stop-line. Platoon sizes in different runs are slightly different since this is also a random variable in the model. The 13 different platoons were then treated in the same way as for the observed pla-

toons and the best K values obtained in this way are presented in table 5.

Simulation	
platoon size	best K
22	0.23
28	0.14
33	0.20
38	0.17
43	0.26
48	0.31
53	0.27
62	0.48
70	0.63
75	0.70
84	0.74
87	0.48
90	0.42

Table 5. Best K values obtained by simulating different platoon-sizes.

These values are represented in figure 12 as a function of platoon size. The shape of the curve is similar to the one obtained in the field study (figure 11), with different maximum values of K due to the different characteristics chosen in the simulation.

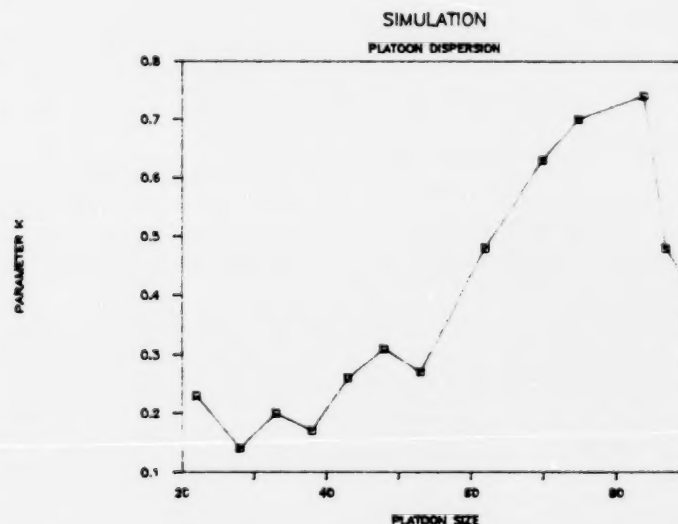


Figure 12. Parameter K for the simulated platoons.

The simulation program produces different useful outputs. One of these is the macroscopic relationship derived from the car following model. Figure 13 gives an example for this output, which could be used for further analysis of the pla-

toon dispersion.

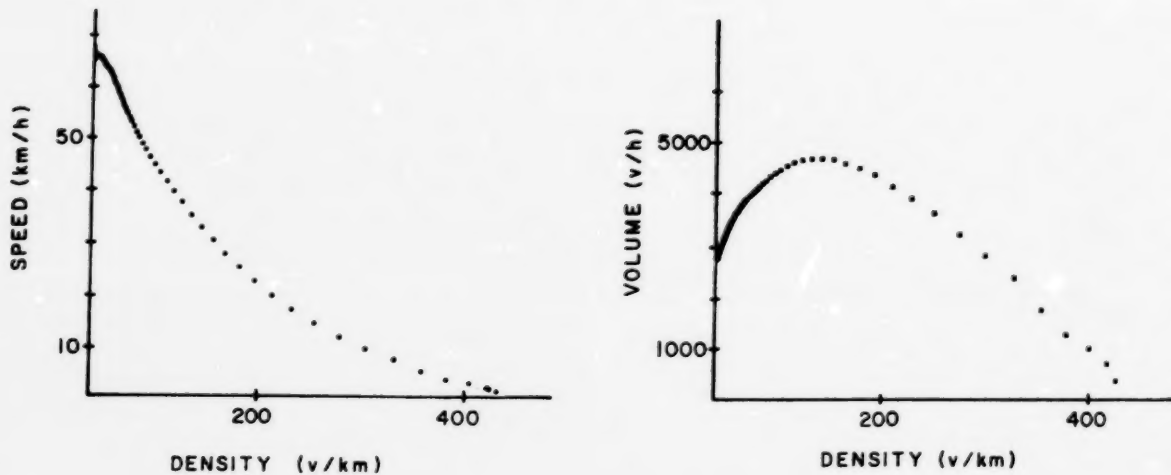


Figure 13. Macroscopic relationship derived from car following model.

Other important outputs are several statistics which were calculated to help in the interpretation of the simulated platoons. One of these is the number of lane changes (from left to right and from right to left). Figure 14 illustrates this variable for the simulated platoons.

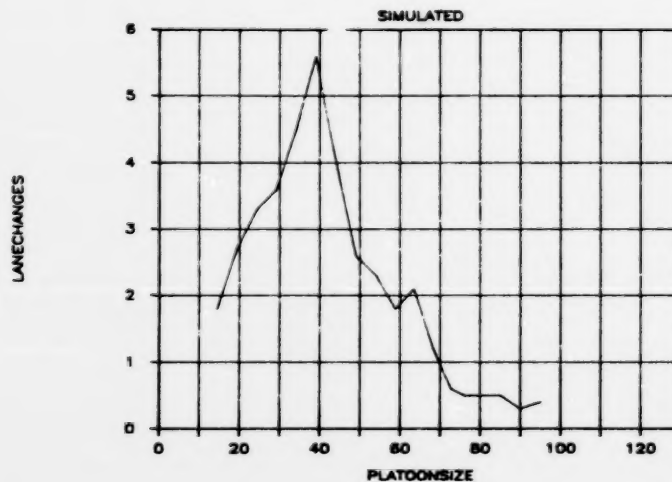


Figure 14. Number of lane changes vs. platoon size.

As volumes increase, drivers lose the ability to maintain their desired speeds.

Drivers have to reduce their speed to those of the slower vehicles or, if possible, change lanes. The lane changing possibility diminishes after a maximum value if volumes are further increased. The maximum value may be interpreted as the point at which drivers become influenced by the presence of vehicles ahead of them. The number of overtaking and passing opportunities can be viewed as an index for friction. Even if the Highway Capacity Manual 1950 (16) gives only an example for 2 lane highways, the form of the relationship in figure 15, which is reproduced here from the manual, may also be valid in multi-lane arterial analysis.

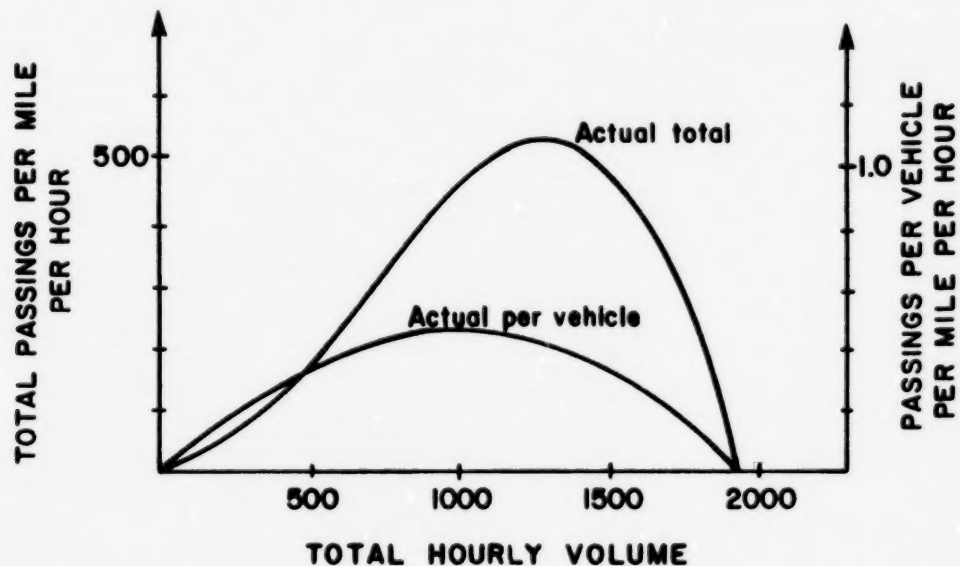


Figure 15. Passing opportunities on a 2 lane highway. HCM 1950

Two other variables produced by the simulation model are of interest, these are the velocity gradient defined by Helly (8) and the acceleration noise. The values of these variables are illustrated in figures 16 and 17.

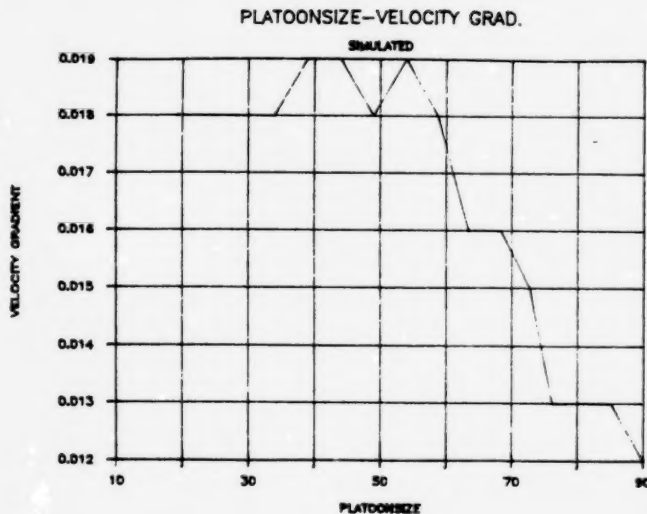


Figure 16. Velocity gradient

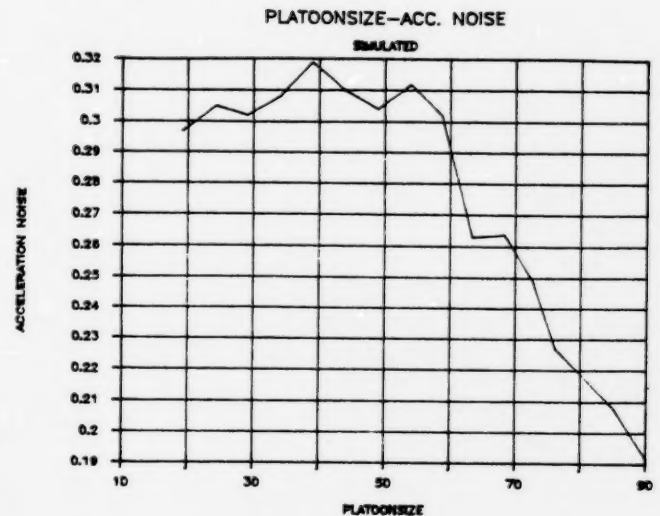


Figure 17. Acceleration noise

The simulated platoons behaved in a way which corresponds fairly well to the proposed relationship between platoon dispersion and traffic volume, since these variables show an increase as volume increases, followed by a subsequent decrease to a minimum value, situated near the capacity.

Conclusion

The degree of platoon dispersion depends on internal and external friction. The external friction is contained in the saturation flow concept. Since the internal friction depends on traffic volumes, the platoon dispersion has to depend also on service volumes. Observed and simulated platoons showed low dispersion at low traffic volumes. External friction being the same, dispersion increases as volumes increase, whereby the actual value of K depends on the kind of arterial studied. Dispersion increases further up to a maximum value as traffic volumes increase. This point of maximum dispersion is observed at volumes of 0.6 to 0.8 of the capacity and is near the point of maximum energy. The diminishing dispersion after further volume increases can be explained by changing driver behavior at or near the point of inflection in the volume-spacing curve.

Further study should be devoted to more platoon observations at different locations and for different lane numbers and cycle lengths in order to define (if possible) a typical curve relating service volumes to dispersion. This relationship could have the following form, where q represents the volumes.

$$F = \frac{1}{1 + (a + bq - cq^2) t}$$

This would enable us to consider platoon dispersion on a link by link basis, which would contribute to improve the results of the traffic signal coordination models.

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